The BFKL Equation at Next-to-Leading Level and Beyond *

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Abstract

On the basis of a renormalization group analysis of the kernel and of the solutions of the BFKL equation with subleading corrections, we propose and calculate a novel expansion of a properly defined effective eigenvalue function. We argue that in this formulation the collinear properties of the kernel are taken into account to all orders, and that the ensuing next-to-leading truncation provides a much more stable estimate of hard Pomeron and of resummed anomalous dimensions.

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The recent calculation [1, 2] of the next-to-leading $\log s$ (NL) corrections to the kernel of the BFKL equation has solved a long-standing problem [3], but has also raised some puzzling questions. In fact, such corrections – which decrease both the gluon anomalous dimension and the hard pomeron intercept – turn out to be so large, that they cannot be taken literally for reasonable values of $\alpha_s \geq 0.1$. The question then arises: what is the origin of such large corrections? and, what use can we make of them?

We do not suggest here to perform exact higher order calculations, not only because they are awkward, but also because they mix with unitarity effects [4] and thus cannot be described within the BFKL equation alone. Our purpose is rather to illustrate the physics of subleading corrections and to perform *partial resummations* on the basis of a renormalization group analysis which improves the perturbative formulation of the small-x problem.

Our argument follows two steps. First, we propose an ansatz for the BFKL solutions which is automatically consistent with the R.G. for a general dependence of the kernel on the running coupling. Secondly, we identify the subleading corrections to be resummed, mainly the scale-dependent ones, as recently suggested [5, 6], and the collinear singular ones [7]. The improved NL expansion follows naturally.

1 Renormalization Group Analysis

In small-x physics ($s \gg Q^2 \gg \Lambda^2$), Regge theory and the renormalization group come to a sort of clash, which provides nontrivial consistency requirements.

It is known [8, 9] that the BFKL equation satisfies R.G. factorization in an asymptotic way. Introducing, by $\boldsymbol{k}$-factorization [10], the NL gluon Green's function

$$\mathcal{G}_\omega(t,t_0) = \left[\omega - \bar{\alpha}_s(t)\big(K_0 + \bar{\alpha}_s(\mu^2)\,K_1\big)\right]^{-1} \quad , \qquad \left(\bar{\alpha}_s = \frac{N_c \alpha_s}{\pi}\right) \quad , \tag{1.1}$$

its asymptotic form for $t \equiv \log \boldsymbol{k}^2/\Lambda^2 \gg t_0$ is given by

$$\mathcal{G}_\omega(t,t_0) \simeq C(\omega, \alpha_s(t)) \left[\exp \int_{t_0}^{t} \gamma_\omega^+(\alpha_s(\tau))\mathrm{d}\tau\right] C_0(\omega, \alpha_s(t_0)) \quad , \tag{1.2}$$

where γ_ω^+ is the larger eigenvalue of the singlet anomalous dimension matrix, defined by the equation

$$\omega = \bar{\alpha}_s(t)\big(\chi_0(\gamma_\omega^+) + \bar{\alpha}_s(\mu^2)\chi_1(\gamma_\omega^+)\big) \quad . \tag{1.3}$$

Solutions to this equation are found so long as $\omega \geq \omega_{\mathbb{P}}(\alpha_s)$, the "hard pomeron" intercept, which in this treatment is estimated from eq. (1.3) and from the NL calculations [2] to be

$$\omega_{\mathbb{P}}(\alpha_s) = \bar{\alpha}_s\big(\chi_0(\tfrac{1}{2}) + \bar{\alpha}_s\chi_1(\tfrac{1}{2})\big) \simeq 2.77\bar{\alpha}_s(1 - 6.47\bar{\alpha}_s) \quad , \tag{1.4}$$

leading to the anomalously large NL corrections mentioned before.

While the t-dependent coefficient $C(\omega, \alpha_s(t))$ in eq. (1.2) is perturbatively calculable, the t_0-dependent one C_0 is not, because it contains the leading ω-singularity, that we call simply the "pomeron". The latter turns out to be *dependent* on how $\alpha_s(t)$ is smoothed out or cut-off around $t = 0$ ($\boldsymbol{k}^2 = \Lambda^2$), in order to avoid the Landau pole. However if $t_0 \gg 1$, it is suppressed by a power of $\Lambda^2/\boldsymbol{k}_0^2$.

Perturbative calculations are thus hampered by two ω-singularities. The first one, the hard pomeron $\omega_{\mathbb{P}}(\alpha_s(t))$ is a singularity of the *anomalous dimension expansion*, not necessarily of the full amplitude, and is our main concern here. It dominates an intermediate-x regime, characterized by quite large anomalous dimensions. The second singularity – the pomeron $\omega_{\mathbb{P}}$ – is the leading ω-singularity of the amplitude and dominates the very small-x regime, but is non-perturbative and is not directly related to scaling violations.

The treatment of the hard pomeron just summarized has, however, the limitation that $\alpha_s(\mu^2)$ in eq. (1.1) is kept frozen in front of K_1, while the overall factor $\alpha_s(t)$ is allowed to run. Even if consistent at NL level, this approach should be improved in order to perform partial resummations to all orders in $\alpha_s(t)$, as we envisage here. How can this be achieved?

Our main proposal is to use ω as the expansion parameter of the BFKL solutions, instead of $\alpha_s(t)$. More precisely, we look for the (regular) solution [9] of the homogeneous BFKL equation

$$\omega\mathcal{F}_\omega(t) = [K_\omega\mathcal{F}_\omega](t) \quad , \tag{1.5}$$

where the kernel K_ω has a general $\alpha_s(t)$-dependence of the form (see sec. (2))

$$K_\omega(\boldsymbol{k},\boldsymbol{k}') = \bar{\alpha}_s(t) K_0^\omega(\boldsymbol{k},\boldsymbol{k}') + \bar{\alpha}_s(t)^2 K_1^\omega(\boldsymbol{k},\boldsymbol{k}') + \cdots \qquad , \qquad t \equiv \log \frac{\boldsymbol{k}^2}{\Lambda^2} \quad , \tag{1.6}$$

and the $K_n^\omega : n = 0, 1, \cdots$ are scale invariant kernels which may be ω-dependent.

We then assume the ansatz

$$\mathcal{F}_\omega(t) = \frac{1}{\boldsymbol{k}^2} \int_{\frac{1}{2}-\mathrm{i}\infty}^{\frac{1}{2}+\mathrm{i}\infty} \frac{\mathrm{d}\gamma}{2\pi\mathrm{i}}\, \mathrm{e}^{\gamma t - \frac{1}{b\omega} X(\gamma,\omega)} \qquad , \qquad b \equiv \frac{\pi}{N_c} \frac{11N_c - 2N_f}{12\pi} \quad , \tag{1.7}$$

where $X(\gamma,\omega)$ is to be found by solving eq. (1.5), as a power series in ω. The relevance of such solution is that it is shown to represent [11] the asymptotic form of the Green's function (1.1) for a general form of the kernel K, as follows

$$\mathcal{G}_\omega(t,t_0) \simeq \mathcal{F}_\omega(t) \tilde{\mathcal{F}}_\omega(t_0) \qquad , \qquad (t \gg t_0 \gtrsim 1) \tag{1.8}$$

where, for a given regularization of the Landau pole, $\mathcal{F}_\omega(t)$ has the form (1.7), while $\tilde{\mathcal{F}}_\omega(t_0)$ contains the (regularization-dependent) pomeron singularity.

While the detailed properties of $X(\gamma,\omega)$ are dependent on how $\alpha_s(t)$ in eq. (1.6) is smoothed out or cut-off around $t = 0$, the large-t behaviour of eq. (1.7) is instead universal, provided a saddle point $\bar{\gamma}_\omega(t)$ of the exponent $E_\omega(\gamma,t)$ of eq. (1.7) exists and is stable. We thus assume, in the asymptotic regime $bt \gtrsim \frac{1}{\omega} \gg 1$, the expansion

$$E_\omega(\gamma,t) = E_\omega(\bar{\gamma}_\omega,t) - \frac{1}{2b\omega}\chi'(\bar{\gamma}_\omega,\omega)(\gamma-\bar{\gamma}_\omega)^2 - \frac{1}{6b\omega}\chi''(\bar{\gamma}_\omega,\omega)(\gamma-\bar{\gamma}_\omega)^3 + \cdots \quad , \tag{1.9}$$

with the saddle point conditions

$$b\omega t \equiv \chi(\bar{\gamma}_\omega(t),\omega) = X'(\bar{\gamma}_\omega(t),\omega) \quad , \quad \chi'(\bar{\gamma}_\omega(t),\omega) < 0 \qquad , \qquad (\#)' \equiv \frac{\partial}{\partial\gamma}(\#) \quad , \tag{1.10}$$

which need to be checked "a posteriori".

The crucial property of the representation (1.7) around the saddle point of eqs. (1.9) and (1.10) is that it takes a form consistent with the R.G., namely

$$\boldsymbol{k}^2 \mathcal{F}_\omega(t) \sim \frac{1}{\sqrt{2\pi\big(-\chi'(\bar{\gamma}_\omega(t),\omega)\big)}} \exp\left\{E_\omega(\bar{\gamma}_\omega(t),t)\right\} \times \big(1 + \mathcal{O}(\omega)\big) \quad , \tag{1.11}$$

where, for any ω-dependence,

$$E_\omega(\bar{\gamma}_\omega(t), t) = \bar{\gamma}_\omega(t)\, t - \frac{1}{b\omega}\, X(\bar{\gamma}_\omega(t), \omega) = \int^t \bar{\gamma}_\omega(\tau) \mathrm{d}\tau + \mathrm{const} \quad , \tag{1.12}$$

so that eq. (1.8) takes the form anticipated in eq. (1.2) for the gluon Green's function.

Furthermore, the form of $\chi(\gamma, \omega)$ can be found, as an expansion in ω, from the original equation (1.5), expanded around the saddle point. For instance, if we let the coupling run up to NL level, by expanding the eigenvalue functions $\chi_0^\omega(\gamma), \chi_1^\omega(\gamma), \cdots$ of $K_0^\omega, K_1^\omega, \cdots$ up to the relevant order in $\gamma - \bar{\gamma}$, we find the solution

$$\chi(\gamma, \omega) = \chi_0^\omega(\gamma) + \omega\, \frac{\chi_1^\omega(\gamma)}{\chi_0^\omega(\gamma)} + \omega^2\, \frac{1}{\chi_0^\omega(\gamma)} \left(\frac{\chi_2^\omega(\gamma)}{\chi_0^\omega(\gamma)} - \left(\frac{\chi_1^\omega(\gamma)}{\chi_0^\omega(\gamma)} \right)^2 + b \left(\frac{\chi_1^\omega(\gamma)}{\chi_0^\omega(\gamma)} \right)' \right) + \mathcal{O}(\omega^3) \tag{1.13}$$

which has been pushed up to NNL level in ω. Some of the terms of the ω-expansion (1.13) follow from the trivial replacement $1/t = b\omega/\chi_0^\omega(\gamma) + \cdots$, while the term $b\big(\chi_1^\omega(\gamma)/\chi_0^\omega(\gamma)\big)'$ needs a careful treatment of fluctuations. This result is confirmed, and extended to higher orders, by the replacement $t \to \partial_\gamma$ in the BFKL equation, which leads to a non linear differential equation for χ [11].

The result (1.13) is the basic formula that we present here, in order to resum some large subleading corrections, and to obtain a smoother NL truncation for the remaining ones. Basically, the coefficients of the ω-expansion (1.13) turn out to be less singular than the ones in the α_s-expansion in the collinear limits $\gamma \to 0$ ($\gamma \to 1$), related to $t \gg t_0$ ($t_0 \gg t$). This is due to the powers of χ_0 in the denominators which damp the collinear behaviour and to cancellations to be explained below. This feature will yield a smoother expansion in the region $\gamma \simeq 1/2$ also, which is the relevant one for the hard pomeron estimate.

2 Improved next-to-leading expansion

Before proceeding further, we need to understand the physics of subleading corrections, in order to single out the ones that need to be resummed. We limit ourselves, for simplicity, to the gluonic contributions, because the $q\bar{q}$ ones turn out to be small [1].

The explicit eigenvalues of the gluonic part of the kernels K_0 and K_1 are [2]

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \underset{\gamma\to 0}{\simeq} \frac{1}{\gamma} - 2\psi(1)\,\gamma^2 + \cdots \tag{2.1}$$

and

$$\begin{aligned}
\chi_1(\gamma) &= \left(-\frac{11}{24}(\chi_0^2(\gamma) + \chi_0^{'}(\gamma))\right) + \left[-\frac{1}{4}\,\chi_0^{''}(\gamma)\right] + \\
&+\frac{1}{4}\left\{\left(\frac{67}{9} - \frac{\pi^2}{3}\right)\chi_0(\gamma) - \left(\frac{\pi}{\sin\pi\gamma}\right)^2 \frac{\cos\pi\gamma}{3(1-2\gamma)}\left(11 + \frac{\gamma(1-\gamma)}{(1+2\gamma)(3-2\gamma)}\right)\right\} + \\
&+\frac{3}{2}\,\zeta(3) + \frac{\pi^3}{4\sin\pi\gamma} - \Phi(\gamma) \quad , \qquad\qquad (2.2)\\
\Phi(\gamma) &\equiv \sum_{n=0}^{\infty}(-)^n\left[\frac{\psi(n+1+\gamma)-\psi(1)}{(n+\gamma)^2} + \frac{\psi(n+2-\gamma)-\psi(1)}{(n+1-\gamma)^2}\right] \quad .
\end{aligned}$$

Here we quote the form of χ_1 referring to the choice $s_0 = kk_0$ of the energy scale in the NL $\boldsymbol{k}$-factorization formula [10]:

$$\frac{\mathrm{d}\sigma^{AB}}{\mathrm{d}^2\boldsymbol{k}\,\mathrm{d}^2\boldsymbol{k}_0} = \int \frac{\mathrm{d}\omega}{2\pi\mathrm{i}}\; h_A(\boldsymbol{k})\, h_B(\boldsymbol{k}_0)\, \mathcal{G}_\omega(\boldsymbol{k},\boldsymbol{k}_0)\left(\frac{s}{kk_0}\right)^{\omega} \quad , \tag{2.3}$$

where $\boldsymbol{k}$ and $\boldsymbol{k}_0$ are the transverse momenta of the outgoing jets in the fragmentation regions of the incoming partons A and B respectively.

In the expansion (2.2) we have singled out some contributions which have a natural physical interpretation, namely the running coupling terms (in round brackets), the scale-dependent terms (in square brackets) and the collinear terms (in curly brackets).

The running coupling terms are proportional to the one-loop beta function coefficient b and refer to the choice of eq. (1.6) of factorizing the running coupling $\alpha_s(t)$ at the upper scale $\boldsymbol{k}^2$ of the kernel $K_\omega(\boldsymbol{k},\boldsymbol{k}^{'})$, instead of a symmetrical combination of $\boldsymbol{k}$ and $\boldsymbol{k}^{'}$. Their explicit form can be shown [6] to shift the effective scale to the symmetrical one $\boldsymbol{q}^2 = (\boldsymbol{k}-\boldsymbol{k}^{'})^2$, which is the one occurring in the phase space $\nu \gtrsim \boldsymbol{q}^2$, where ν is the longitudinal part of the two-gluon subenergy in the s-channel. In γ-space, the running coupling terms show a double pole at $\gamma = 1$ only.

The collinear and scale-dependent terms have multiple poles at both $\gamma = 0$ and $\gamma = 1$ due to the behaviour of the kernel for $k \gg k^{'}$ and $k^{'} \gg k$. All these poles are actually of

collinear origin, and the cubic ones are dependent on the choice of the scale of the energy in eq. (2.3), whether it is $s_0 = kk_0$ (the symmetrical choice), or instead $s_1 = k^2$ ($s_2 = k_0^2$), which is the choice leading to the correct scaling variable k^2/s (k_0^2/s) for $k \gg k_0$ ($k_0 \gg k$).

It has been already remarked [12] that the choice $s_0 = kk_0$ is not collinear safe and leads to the nasty singularities of type $1/\gamma^3$ $(1/(1-\gamma)^3)$ of χ_1 in eq. (2.2), which correspond to the double logarithmic behaviour $\sim \log^2 k/k'$ of the kernel when $k \gg k'$ ($k' \gg k$).

On the other hand, complete information about the collinear behaviour of the kernel in such regions comes from the renormalization group equations. In general the kernel $K_\omega(t,t')$ in eq. (1.6) is collinear finite and symmetrical in its arguments for $s_0 = kk_0$, so that it must have the form

$$K_\omega(\alpha_s(\mu^2), \mu^2, \boldsymbol{k}, \boldsymbol{k}') = \frac{\bar{\alpha}_s(t)}{\boldsymbol{k}^2} \hat{K}_\omega(\alpha_s(t); t, t') = \frac{\bar{\alpha}_s(t')}{\boldsymbol{k}'^2} \hat{K}_\omega(\alpha_s(t'); t', t) \quad , \tag{2.4}$$

where $\hat{K}$ can be expanded in $\alpha_s(t)$ with scale-invariant coefficients, as assumed in eq. (1.6).

Furthermore, calling $K_\omega^{(1)}$ ($K_\omega^{(2)}$) the kernel at energy scale k^2 (k_0^2), it acquires for $k'/k \to 0$ ($k/k' \to 0$) the collinear singularities due to the non singular part of the gluon anomalous dimension in the Q_0-scheme [13] which, neglecting the $q\bar{q}$ part, is

$$\begin{aligned} \tilde{\gamma}(\omega) &= \gamma_{gg}(\omega) - \frac{\bar{\alpha}_s}{\omega} = \bar{\alpha}_s A_1(\omega) + \bar{\alpha}_s^2 A_2(\omega) + \cdots \quad , \\ A_1(\omega) &= -\frac{11}{12} + \mathcal{O}(\omega) \quad , \quad A_2(\omega) = 0 + \mathcal{O}(\omega) \quad , \end{aligned} \tag{2.5}$$

the singular part being taken into account by the BFKL iteration itself. It follows that, for $k \gg k'$,

$$\begin{aligned} K_\omega^{(1)}(\alpha_s(t); t, t') &\simeq \frac{\bar{\alpha}_s(t)}{\boldsymbol{k}^2} \ \exp \int_{t'}^{t} \tilde{\gamma}(\omega, \alpha_s(\tau)) \, \mathrm{d}\tau \\ &\simeq \frac{\bar{\alpha}_s(t)}{\boldsymbol{k}^2} \left(1 - b\bar{\alpha}_s(t) \log \frac{\boldsymbol{k}^2}{\boldsymbol{k}'^2}\right)^{-\frac{A_1(\omega)}{b}} \quad , \end{aligned} \tag{2.6}$$

with a similar behaviour, with t and t' interchanged, for $K_\omega^{(2)}$ in the opposite limit $k' \gg k$.

In order to derive from eq. (2.6) the γ-dependent singularities of K_ω care should be taken of the relationships [12]

$$K_\omega(\boldsymbol{k}, \boldsymbol{k}') = \left(\frac{k'}{k}\right)^\omega K_\omega^{(1)}(\boldsymbol{k}, \boldsymbol{k}') = \left(\frac{k}{k'}\right)^\omega K_\omega^{(2)}(\boldsymbol{k}, \boldsymbol{k}') \tag{2.7}$$

which shift the γ-singularities of $K_\omega^{(1)}$ ($K_\omega^{(2)}$) by $-\omega/2$ ($+\omega/2$). As a consequence, by eq. (2.6), the γ-singularities of the eigenvalue functions χ_n^ω of the kernels K_n^ω in eq. (1.6) are

$$\begin{aligned} \chi_n^\omega(\gamma) &\simeq \frac{1\cdot A_1(A_1+b)\cdots(A_1+(n-1)b)}{(\gamma+\frac{1}{2}\omega)^{n+1}} \quad , \quad (\gamma \ll 1) \\ &\simeq \frac{1\cdot(A_1-b)(A_1-2b)\cdots(A_1-nb)}{(1-\gamma+\frac{1}{2}\omega)^{n+1}} \quad , \quad (1-\gamma\ll 1) \quad . \end{aligned} \tag{2.8}$$

In particular, χ_0^ω has symmetrical singularities and χ_1^ω slightly asymmetrical ones, as follows:

$$\chi_0^\omega(\gamma) \simeq \frac{1}{\gamma+\frac{1}{2}\omega} + \frac{1}{1-\gamma+\frac{1}{2}\omega} \quad , \tag{2.9}$$

$$\chi_1^\omega(\gamma) \simeq \frac{A_1}{(\gamma+\frac{1}{2}\omega)^2} + \frac{A_1-b}{(1-\gamma+\frac{1}{2}\omega)^2} \quad . \tag{2.10}$$

The shift by $\pm\omega/2$ in eqs. (2.8)-(2.10) already represents a resummation of the scale-dependent singularities, as noticed by Salam [5], and it provides the correct scale-dependent terms ($\sim 1/\gamma^3$, $1/(1-\gamma)^3$) by the ω-expansion of eq. (2.9). On the other hand, eq. (2.10) provides the correct collinear behaviour of $K_\omega^{(1)}$ ($K_\omega^{(2)}$) close to $\gamma=0$ ($\gamma=1$) at NL order, and eq. (2.8) generalizes it to all orders.

The simplest realization of eq. (2.9) is to assume, following ref. [5], that χ_0^ω in eqs. (1.6) and (1.13) is provided by the eigenvalue function of the Lund model [14]

$$\chi_0^\omega(\gamma) = \psi(1) - \psi(\gamma+\tfrac{1}{2}\omega) + \psi(1) - \psi(1-\gamma+\tfrac{1}{2}\omega) \quad , \tag{2.11}$$

which corresponds to the kernel

$$K_0^\omega(\boldsymbol{k},\boldsymbol{k}') = K_0(\boldsymbol{k},\boldsymbol{k}')\left(\frac{k_<}{k_>}\right)^\omega \qquad , \qquad k_< \equiv \min\{k,k'\} \quad , \quad k_> \equiv \max\{k,k'\} \quad . \tag{2.12}$$

Here the ω-dependent factor $(k_</k_>)^\omega$ can be directly interpreted as an s-channel threshold factor [15] in properly defined Toller variables [16].

The NL terms in eq. (1.13) are now simply identified from eq. (2.2) after subtraction of the singular part of the scale-dependent terms already taken into account in eq. (2.11).

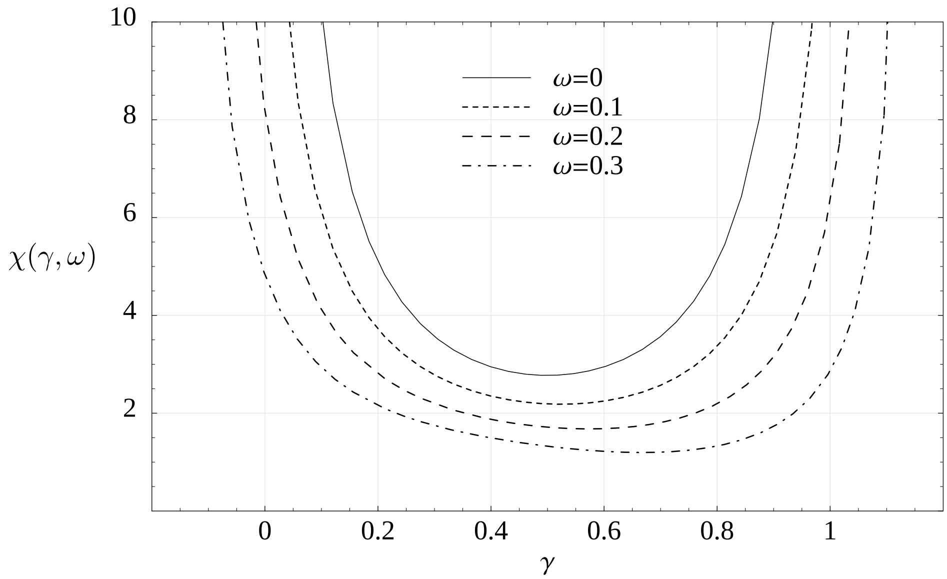

Figure 1: *Resummed eigenvalue function* $\chi(\gamma, \omega)$ *for various values of* ω.

This subtraction eliminates the cubic singularities in χ_1, but leaves double and single poles which need to be shifted by the procedure of ref. [5]. This amounts to the replacement of χ_1 by

$$\begin{aligned} \chi_1^\omega(\gamma) \equiv \chi_1(\gamma) &+ \frac{1}{2}\chi_0(\gamma)\frac{\pi^2}{\sin^2(\pi\gamma)} + \frac{\pi^2}{6}\left(\chi_0^\omega(\gamma) - \chi_0(\gamma)\right) + A_1(\omega)\psi'(\gamma + \tfrac{1}{2}\omega) \\ &- A_1(0)\psi'(\gamma) + (A_1(\omega) - b)\psi'(1 - \gamma + \tfrac{1}{2}\omega) - (A_1(0) - b)\psi'(1 - \gamma) \\ \equiv A_1(\omega)\psi'(\gamma + \tfrac{1}{2}\omega) &+ (A_1(\omega) - b)\psi'(1 - \gamma + \tfrac{1}{2}\omega) + \frac{\pi^2}{6}\,\chi_0^\omega(\gamma) + \tilde{\chi}_1(\gamma) \quad , \end{aligned} \tag{2.13}$$

where now $\tilde{\chi}_1(\gamma)$ has no $\gamma = 0$ nor $\gamma = 1$ singularities at all.

What about higher subleading orders in eq. (1.13)? Here the crucial observation is that, after substitution of the behaviour (2.8) in the NNL term of eq. (1.13), the leading γ-singularities *cancel out close to both* $\gamma = 0$ *and* $\gamma = 1$. Therefore, the coefficient of ω^2 in eq. (1.13) has *no* γ*-singularities* at all, and no further resummation is needed. This peculiar feature, confirmed at higher orders [11], is due to the fact that running coupling effects are already taken into account by the representation (1.7) with the solution (1.13).

Our truncated NL effective eigenvalue function (fig. 1) then reads

$$\chi(\gamma, \omega) = \chi_0^\omega(\gamma) + \omega\,\frac{\chi_1^\omega(\gamma)}{\chi_0^\omega(\gamma)} + \mathcal{O}(\omega^2) \quad , \tag{2.14}$$

and contains only the shifted single poles in γ, while the neglected terms have no leading twist γ-singularities. This expression performs the resummation of the scale-dependent singularities proposed in ref. [5], but it also takes into account the collinear singularities of eq. (2.8) to all orders. This resummation effect is perhaps more easily seen by using eq. (2.14) in order to recast eq. (1.10) in the form

$$b\omega t = \chi_0^\omega \left[1 - \frac{1}{bt} \left(\frac{\chi_1^\omega}{\chi_0^\omega} + \mathcal{O}(\omega) \right) \right]^{-1} \quad , \tag{2.15}$$

which defines an effective χ-function as a power series in α_s. By specializing eq. (2.15) to the scale $\boldsymbol{k}^2$ (shift $\gamma + \omega/2 \to \gamma$), it is apparent that all the small γ poles $(A_1(\omega)/\gamma bt)^n$ are resummed as a geometric series, to provide the full one-loop anomalous dimension of eq. (2.5).

The expression (2.14) for $\chi(\gamma, \omega)$ is not fully symmetrical under the replacement $\gamma \leftrightarrow (1-\gamma)$, but keeps the asymmetrical $-b\omega\chi_0'/2\chi_0$ contribution of eq. (2.2). The breakdown of the $\gamma \leftrightarrow (1-\gamma)$ symmetry in eq. (1.7) follows from that of scale invariance by running coupling effects. It is easy to recognize that the contribution $-b\omega\chi_0'/2\chi_0$ to $\chi(\gamma, \omega)$ yields a factor $\sqrt{\chi_0^\omega(\gamma)}$ in the representation (1.7), and this factor *is required* [11] by the continuum normalization of the eigenfunctions of K_ω provided by the Jacobian $X'(\gamma, \omega) = \chi_0^\omega(\gamma) + \mathcal{O}(\omega)$. Therefore, even if the kernel K_ω and the Green's function $\mathcal{G}_\omega$ are symmetrical operators, the effective eigenvalue function $\chi(\gamma, \omega)$ must have some asymmetry, due to running coupling effects.

It appears from fig. (1) that $\chi(\gamma, \omega)$, with the b-dependent asymmetry pointed out before, is a decreasing function of ω because of the negative sign of NL corrections, and contains a stable minimum for all the relevant ω values. This means that in the present approach the hard pomeron can be estimated up to sizeable values of $\alpha_s = 0.2 \div 0.3$, and it turns out to be $\omega_{\mathbb{P}} = 0.26 \div 0.32$ in this range, which is a reasonable value for the HERA data [17] at intermediate values or Q^2.

Since in the expansion of eq. (2.14) the neglected terms – of order ω^2 or higher – have no γ-singularities, they only affect the evaluation of eq. (1.10) by a roughly γ-independent uncertainty which corresponds to a change of scale $\Delta(bt) = \mathcal{O}(\omega)$, or $\Delta\alpha_s = \mathcal{O}(\omega)\alpha_s^2$. This

means that the error affecting the NL truncation is uniformly of NNL order for all values of γ – whether $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s/\omega)$ or $\mathcal{O}(1)$ – and therefore it cannot change too much the $\omega_{\mathbb{P}}$-estimate given above.

A detailed analysis of the anomalous dimensions and of the hard pomeron properties in the present approach is left to a parallel investigation [11].

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